

# Utility-Preserving Differentially Private Data Releases Via Individual Ranking Microaggregation

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## Abstract

Being able to release and exploit open data gathered in information systems is crucial for researchers, enterprises and the overall society. Yet, these data must be anonymized before release to protect the privacy of the subjects to whom the records relate. Differential privacy is a privacy model for anonymization that offers more robust privacy guarantees than previous models, such as  $k$ -anonymity and its extensions. However, it is often disregarded that the utility of differentially private outputs is quite limited, either because of the amount of noise that needs to be added to obtain them or because utility is only preserved for a restricted type and/or a limited number of queries. On the contrary,  $k$ -anonymity-like data releases make no assumptions on the uses of the protected data and, thus, do not restrict the number and type of doable analyses. Recently, some authors have proposed mechanisms to offer general-purpose differentially private data releases. This paper extends such works with a specific focus on the preservation of the utility of the protected data. Our proposal builds on microaggregation-based anonymization, which is more flexible and utility-preserving than alternative anonymization methods used in the literature, in order to reduce the amount of noise needed to satisfy differential privacy. In this way, we improve the utility of differentially private data releases. Moreover, the noise reduction we achieve does not depend on the size of the data set, but just on the number of attributes to be protected, which is a more desirable behavior for large data sets. The utility benefits brought by our proposal are empirically evaluated and compared with related works for several data sets and metrics.

*Keywords:* Privacy-preserving data publishing, Differential privacy,

## 1. Introduction

Releasing and exploiting open data is crucial to boost progress of knowledge, economy and society. Indeed, the availability of such data facilitates research and allows better marketing, better planning and better social services. However, data publication often faces privacy threats due to the confidentiality of the information that is released for secondary use. To tackle this problem, a plethora of methods aimed at data anonymization have been proposed within the field of statistical disclosure control [1]. Such methods distort input data in different ways (*e.g.* noise addition, removal, sampling, data generalization, etc.) so that the probability of re-identifying individuals and, thus, disclosing their confidential information is brought below a tolerable threshold. Even though those methods have been shown to improve privacy protection while preserving a reasonable level of analytical utility (the main motivation of data publishing), they offer no formal privacy guarantees.

In contrast, privacy models proposed in recent years within the computer science community [2, 3] seek to attain a predefined notion of privacy, thus offering *a priori* privacy guarantees. These guarantees are interesting because they ensure a minimum level of privacy regardless of the type of transformation performed on input data. Among such models, *k*-anonymity and the more recent  $\epsilon$ -differential privacy have received a lot of attention.

*k*-Anonymity [4, 5] seeks to make each record in the input data set indistinguishable from, at least,  $k - 1$  other records, so that the probability of re-identification of individuals is, at most,  $1/k$ . Different anonymization methods have been proposed to achieve that goal, such as removal of outlying records, generalization of values to a common abstraction [5, 6, 7, 8] or multivariate microaggregation [9, 10]. The latter method partitions a data set into groups of at least  $k$  similar records and replaces the records in each group by a prototypical record (*e.g.* the centroid record, that is, the average record). Whatever the computational procedure, *k*-anonymity focuses on masking quasi-identifier attributes; these are attributes (*e.g.*, Age, Gender, Zipcode and Race) that are assumed to enable re-identifying the respondent of a record because they are linkable to analogous attributes available in external identified data sources (like electoral rolls, phone books, etc.). *k*-Anonymity does not mask confidential attributes (*e.g.*, salary, health condition, political preferences, etc.) unless they are also quasi-identifiers. While

$k$ -anonymity has been shown to provide reasonably useful anonymized results, especially for small  $k$ , it is also vulnerable to attacks based on the possible lack of diversity of the non-anonymized confidential attributes or on additional background knowledge available to the attacker [11, 12, 13, 14].

Unlike  $k$ -anonymity, the more recent  $\epsilon$ -differential privacy [15] model does not make any assumptions on which attributes are quasi-identifiers, that is, on the background knowledge available to potential attackers seeking to re-identify the respondent of a record.  $\epsilon$ -Differential privacy guarantees that the anonymized output is insensitive (up to a factor dependent on  $\epsilon$ ) to the modification, deletion or addition of any single input record in the original data set. In this way, the privacy of any individual is not compromised by the publication of the anonymized output, which is a much more robust guarantee than the one offered by  $k$ -anonymity. Differential privacy is attained by adding an amount of noise to the original outputs that is proportional to the *sensitivity* of such outputs to modifications of particular individual records in the original data set. This sensitivity does not depend on the specific values of attributes in specific records, but on the domains of those attributes. Basing the sensitivity and hence the added noise on attribute domains rather than attribute values satisfies the privacy guarantee but may yield severely distorted anonymized outputs, whose utility is very limited. Because of this,  $\epsilon$ -differential privacy was originally proposed for the *interactive* scenario, in which the outputs are the answers to interactive queries rather than the data set itself. When applying  $\epsilon$ -differential privacy to this scenario, the anonymizer returns noise-added answers to interactive queries. In this way, the accuracy/utility of the response to a query depends on the sensitivity of the query, which is usually less than the sensitivity of the data set attributes. However, the interactive setting of  $\epsilon$ -differential privacy limits the number and type of queries that can be performed. The proposed extensions of  $\epsilon$ -differential privacy to the *non-interactive* setting (generation of entire anonymized data sets) overcome the limitation on the number of queries, but not on the type of queries for which some utility is guaranteed (see Section 2.2 below).

### 1.1. Contribution and plan of this paper

In previous works [16, 17], we showed that the noise required to fulfill differential privacy in the non-interactive setting can be reduced by using a special type of microaggregation-based  $k$ -anonymity on the input data set. The rationale is that the microaggregation performed to achieve  $k$ -anonymity helps reducing the sensitivity of the input versus modifications of individual records. As a result, data utility preservation can be improved

(in terms of less data distortion) without renouncing the strong privacy guarantee of differential privacy. With such a mechanism, the sensitivity reduction depends on the number of  $k$ -anonymized groups to be released; in turn, this number is a function of the value of  $k$  and the cardinality of the data set. The larger the group size  $k$ , the less sensitive are the group centroids resulting from the microaggregation; on the other hand, the smaller the data set, the smaller the number of different group centroids in the microaggregated data set. Thus, as the group size increases or the data set size decreases, the sensitivity decreases and less noise needs to be added to reach differential privacy. Hence, the resulting differentially private data have higher utility. In the two abovementioned works, we empirically showed that the noise reduction more than compensates the information loss introduced by microaggregation.

In line with [16, 17], in this paper we investigate other transformations of the original data aimed at reducing their sensitivity. The proposal in this paper is based on individual ranking microaggregation, a kind of microaggregation that is more flexible and utility-preserving than the one used in [16, 17]. In contrast to the previous works, the reduction of sensitivity achieved by the method presented in this paper does not depend on the size of the data set, but just on the number of attributes to be protected. This is more desirable for large data sets or in scenarios in which only the confidential attributes should be protected. In fact, experiments carried on two reference data sets show a significant improvement of data utility (in terms of relative error and preservation of attribute distributions) with respect to the previous work. Moreover, the microaggregation mechanism used in this paper is simpler and more scalable, which facilitates implementation and practical deployment.

The rest of this paper is organized as follows. Section 2 reviews background on microaggregation,  $\epsilon$ -differential privacy and  $\epsilon$ -differentially private data publishing. Section 3 proposes the new method to generate  $\epsilon$ -differentially private data sets that uses a special type of microaggregation to reduce the amount of required noise. Implementation details are given for data sets with numerical and categorical attributes. Section 4 reports an empirical comparison of the proposed method and previous proposals, based on two reference data sets. The final section gathers some conclusions.

## 2. Background and related work

### 2.1. Background on microaggregation

Microaggregation [9, 18] is a family of anonymization algorithms that works in two stages:

- First, the data set is clustered in such a way that: i) each cluster contains at least  $k$  elements; ii) elements within a cluster are as similar as possible.
- Second, each element within each cluster is replaced by a representative of the cluster, typically the centroid value/tuple.

Depending on whether they deal with one or several attributes at a time, microaggregation methods can be classified into univariate and multivariate:

- Univariate methods deal with multi-attribute data sets by microaggregating one attribute at a time. Input records are sorted by the first attribute, then groups of successive  $k$  values of the first attribute are created and all values within that group are replaced by the group representative (*e.g.* centroid). The same procedure is repeated for the rest of attributes. Notice that all attribute values of each record are moved together when sorting records by a particular attribute; hence, the relation between the attribute values within each record is preserved. This approach is known as *individual ranking* [18, 19] and, since it microaggregates one attribute at a time, its output is not  $k$ -anonymous at the record level. Individual ranking just reduces the variability of attributes, thereby providing some anonymization. In [20] it was shown that individual ranking causes low information loss and, thus, its output better preserves analytical utility. However, the disclosure risk in the anonymized output remains unacceptably high [21].
- To deal with several attributes at a time, the trivial option is to map multi-attribute data sets to univariate data by projecting the former onto a single axis (*e.g.* using the sum of z-scores or the first principal component, see [19]) and then use univariate microaggregation on the univariate data. Another option avoiding the information loss due to single-axis projection is to use *multivariate microaggregation* able to deal with unprojected multi-attribute data [9, 22]. If we define optimal microaggregation as finding a partition in groups of size at least  $k$  such that within-groups homogeneity is maximum, it turns out that,

while optimal univariate microaggregation can be solved in polynomial time [23], unfortunately optimal multivariate microaggregation is NP-hard [24]. This justifies the use of heuristic methods for multivariate microaggregation, such as the MDAV algorithm [10], which has been extensively used to enforce  $k$ -anonymity at the record level [10, 25, 26]. In any case, multivariate microaggregation leads to higher information loss than individual ranking [20].

## 2.2. Background on differential privacy

Differential privacy was originally proposed by [15] as a privacy model in the interactive setting, that is, to protect the outcomes of queries to a database. The assumption is that an anonymization mechanism sits between the user submitting queries and the database answering them.

**Definition 1 ( $\varepsilon$ -Differential privacy).** *A randomized function  $\kappa$  gives  $\varepsilon$ -differential privacy if, for all data sets  $X_1, X_2$  such that one can be obtained from the other by modifying a single record, and all  $S \subset \text{Range}(\kappa)$ , it holds*

$$\Pr(\kappa(X_1) \in S) \leq \exp(\varepsilon) \times \Pr(\kappa(X_2) \in S). \quad (1)$$

The computational mechanism to attain  $\varepsilon$ -differential privacy is often called  $\varepsilon$ -differentially private *sanitizer*. A usual sanitization approach is noise addition: first, the real value  $f(X)$  of the response to a certain user query  $f$  is computed, and then a random noise, say  $Y(X)$ , is added to mask  $f(X)$ , that is, a randomized response  $\kappa(X) = f(X) + Y(X)$  is returned. To generate  $Y(X)$ , a common choice is to use a Laplace distribution with zero mean and  $\Delta(f)/\varepsilon$  scale parameter, where:

- $\varepsilon$  is the differential privacy parameter;
- $\Delta(f)$  is the  $L_1$ -sensitivity of  $f$ , that is, the maximum variation of the query function between neighbor data sets, *i.e.*, sets differing in at most one record.

Specifically, the density function of the Laplace noise is

$$p(x) = \frac{\varepsilon}{2\Delta(f)} e^{-|x|/\Delta(f)}.$$

Notice that, for fixed  $\varepsilon$ , the higher the sensitivity  $\Delta(f)$  of the query function  $f$ , the more Laplace noise is added: indeed, satisfying the  $\varepsilon$ -differential privacy definition (Definition 1) requires more noise when the query function

$f$  can vary strongly between neighbor data sets. Also, for fixed  $\Delta(f)$ , the smaller  $\varepsilon$ , the more Laplace noise is added: when  $\varepsilon$  is very small, Definition 1 almost requires that the probabilities on both sides of Equation (1) be equal, which requires the randomized function  $\kappa(\cdot) = f(\cdot) + Y(\cdot)$  to yield very similar results for all pairs of neighbor data sets; adding a lot of noise is a way to achieve this.

Differential privacy was also proposed for the non-interactive setting in [27, 28, 29, 30]. Even though a non-interactive data release can be used to answer an arbitrarily large number of queries, in all these proposals, this is obtained at the cost of offering utility guarantees only for a restricted class of queries [27], typically count queries.

### 2.3. Related work on differentially private data publishing

In contrast to the general-purpose data publication offered by  $k$ -anonymity, which makes no assumptions on the uses of published data and does not limit the type and number of analyses that can be performed,  $\varepsilon$ -differential privacy severely limits data uses. Indeed, in the interactive scenario,  $\varepsilon$ -differential privacy allows only a limited and pre-selected number of queries of a certain type to be answered; in the extensions to the non-interactive scenario, any number of queries can be answered, but utility guarantees are only offered for a restricted class of queries. We next review the literature on non-interactive  $\varepsilon$ -differential privacy, which is the focus of this paper.

The usual approach to releasing differentially private data sets is based on histogram queries [31, 32], that is, on approximating the data distribution by partitioning the data domain and counting the number of records in each partition set. To prevent the counts from leaking too much information they are computed in a differentially private manner. Apart from the counts, partitioning can also reveal information. One way to prevent partitioning from leaking information consists in using a predefined partition that is independent of the actual data under consideration (*e.g.* by using a grid [33]).

The accuracy of the approximation obtained via histogram queries depends on the size of the histogram bins (the greater they are, the more imprecise is the attribute value) as well as on the number of records contained in them (the more records, the less relative error). For data sets with sparsely populated regions, using a predefined partition may be problematic. Several strategies have been proposed to improve the accuracy of differentially private count (histogram) queries, which we next review. In [34] consistency constraints between a set of queries are exploited to increase accuracy. In [35] a wavelet transform is performed on the data and

noise is added in the frequency domain. In [32, 36] the histogram bins are adjusted to the actual data. In [37], the authors consider differential privacy of attributes whose domain is ordered and has moderate to large cardinality (*e.g.* numerical attributes); the attribute domain is represented as a tree, which is decomposed in order to increase the accuracy of answers to count queries (multi-dimensional range queries). In [38], the authors generalize similar records by using coarser categories for the classification attributes; this results in higher counts of records in the histogram bins, which are much larger than the noise that needs to be added to reach differential privacy. For data sets with a significant number of attributes, attaining differential privacy while at the same time preserving the accuracy of the attribute values (by keeping the histogram bins small enough) becomes a complex task. Observe that, given a number of bins per attribute, the total number of bins grows exponentially with the number of attributes. Thus, in order to avoid obtaining too many sparsely populated bins, the number of bins per attribute must be significantly reduced (with the subsequent accuracy loss). An interesting approach to deal with multidimensional data is proposed in [39, 40]. The goal of these papers is to compute differentially private histograms independently for each attribute (or jointly for a small number of attributes) and then try to generate a joint histogram for all attributes from the partial histograms. This was done for a data set of commuting patterns in [39] and for an arbitrary data set in [40]. In particular, [40] first tried to build a dependency hierarchy between attributes. Intuitively, when two attributes are independent, their joint histogram can be reconstructed from the histograms of each of the attributes; thus, the dependency hierarchy helps determining which marginal or low-dimension histograms are more interesting to approximate the joint histogram. The approaches in [39, 40] can be seen as complementary to our proposal, which can be used as an alternative for computing the histograms. This does not mean that our proposal is not competitive against [39, 40] in terms of data utility. The data utility depends highly on the actual data that we have. The most favorable cases for [40] are data sets with low dependency between attributes (for instance, when attributes are completely independent, computing the marginal histograms is enough) or at least with attributes that depend on a small number of other attributes (for example, for an attribute that depends only on another attribute, the joint histogram for these two attributes is enough). On the contrary, if an attribute has a sizeable and similar level of dependency on all the other attributes, there is no advantage in using [40].

Our work differs from all previous ones in that it is not limited to his-

togram queries and it allows dealing with any type of attributes (ordered or unordered).

#### 2.4. Related work on microaggregation-based differentially-private data publishing

In [17] we presented an approach that combines  $k$ -anonymity and  $\varepsilon$ -differential privacy in order to reap the best of both models: namely, the reasonably low information loss incurred by  $k$ -anonymity and its lack of assumptions on data uses, as well as the robust privacy guarantees offered by  $\varepsilon$ -differential privacy. In that work, we first defined the notion of insensitive microaggregation, which is a multivariate microaggregation procedure that partitions data in groups of  $k$  records with a criterion that is relatively insensitive to changes in the data set. To do so, insensitive microaggregation defines a total order for the joint domains of all the attributes of the input data set  $X$ . Insensitive microaggregation ensures that, for every pair of data sets  $X$  and  $X'$  differing in a single record, the resulting clusters will differ at most in a single record. Hence, the centroids used to replace records of each cluster will have low sensitivity to changes of one input record. Specifically, when centroids are computed as the arithmetic average of the elements of the cluster, the sensitivity is as low as  $\Delta(X)/k$ , where  $\Delta(X)$  is the distance between the most distant records of the joint domains of the input data and  $k$  is the size of the clusters. This sensitivity is much lower and restrained than the one offered by standard microaggregation algorithms, such as MDAV [10], whose output is highly dependent on the input data (*i.e.* modification of a single record may lead to completely different clusters). Note also that the sensitivity of individual input records (*i.e.* without microaggregation) is  $\Delta(X)$ . The downside of insensitive microaggregation is that it yields worse within-cluster homogeneity than standard microaggregation and, hence, higher information loss.

As a result of insensitive microaggregation with cluster cardinality  $k$ , input data are  $k$ -anonymized in such a way that all attributes are considered as being quasi-identifiers. To obtain a differentially private output, an amount of noise needs to be added to cluster centroids that depends on their sensitivity. Centroids provided by insensitive microaggregation have a low sensitivity and thus require little noise, which in turn means incurring low information loss. In data publishing,  $n/k$  centroids are released, each one computed on a cluster of cardinality  $k$  and having sensitivity  $\Delta(X)/k$  (see discussion above). Hence, the sensitivity of the whole data set to be released is  $n/k \times \Delta(X)/k$ . Thus, for numerical data sets, Laplace noise with scale

parameter  $(n/k \times \Delta(X)/k)/\varepsilon$  must be added to each centroid to obtain a  $\varepsilon$ -differentially private output.

For the above procedure to effectively reduce the noise added to the output with respect to standard differential privacy via noise addition with no prior microaggregation, the sensitivity  $n/k \times \Delta(X)/k$  needs to be smaller than the sensitivity of the data set without prior microaggregation, that is,  $\Delta(X)$ . To that end, the  $k$ -anonymity parameter should be adjusted. Increasing  $k$  has two effects: it reduces the contribution of each record to the cluster centroid (thereby reducing the centroid sensitivity), and it reduces the number of clusters (thereby reducing the number of published centroids). Specifically, for  $n/k \times \Delta(X)/k$  to be less than  $\Delta(X)$ , a value  $k > \sqrt{n}$  is needed. This shows that the utility benefits of this method depend on the size  $n$  of the data set.

Here one must acknowledge that, while prior microaggregation enables noise reduction as discussed above, microaggregating records into centroids also entails some utility loss. However, this loss is more than compensated by the benefits brought by cluster centroids being less sensitive than individual records. This is so because microaggregation can exploit the underlying structure of data and reduce the sensitivity with relatively little utility loss. This hypothesis is empirically supported by the extensive evaluations performed in [17, 16].

Even though this previous work effectively reduces the amount of Laplace noise to be added to reach  $\varepsilon$ -differential privacy, the fact that it requires using a microaggregation parameter  $k$  that depends on the number of records  $n$  of the input data set may be problematic for large data sets. In other words, for large data sets, a value of  $k$  so large may be required that the utility loss incurred in the prior microaggregation step cancels the utility gain due to subsequent noise reduction.

To circumvent this problem, in this paper we present an alternative procedure that offers utility gains with respect to standard differential privacy regardless of the number of records of the input data set. Specifically, our method rests on individual ranking univariate (rather than insensitive multivariate) microaggregation to reduce sensitivity in a way that only depends on the number of attributes to be protected.<sup>1</sup> This behavior is more de-

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<sup>1</sup>Microaggregation is used to improve the utility of the released data, while privacy guarantees are provided by differential privacy alone. However, it is interesting to note that, while the multivariate microaggregation in [16, 17] yielded an intermediate  $k$ -anonymous data set, the individual ranking microaggregation in this paper yields an intermediate probabilistically  $k$ -anonymous [41] data set.

sirable in at least the following cases: i) data sets with a large number of records; ii) data sets with a small number of attributes; iii) data sets in which only the confidential attributes, which usually represent a small fraction of the total attributes, should be protected.

### 3. Differential privacy via individual ranking microaggregation

In this section we present a method to obtain differentially private data releases which, for specific data sets, may reduce noise even more than the above-mentioned approach based on multivariate microaggregation. First, we discuss in detail the limitations of that previous mechanism. Then, we present a new proposal that, based on individual ranking, reduces the sensitivity of the microaggregated output independently of the number of records. For simplicity, we first assume data sets with numerical attributes to which an amount of Laplace noise is added to satisfy differential privacy. At the end of this section, we detail how the proposed method can be adapted to deal also with categorical attributes.

#### 3.1. Limitations of multivariate microaggregation for differentially private data releases

In [16, 17], the utility gain was limited by the insensitive multivariate microaggregation. Such microaggregation anonymizes the input data set at the record level.

The sensitivity of the set of  $n/k$  centroids thus obtained is  $n/k \times \Delta(X)/k$  because, in the worst case:

- Changing a single record in the input data set can cause all  $n/k$  clusters to change by one record
- The record changed within each cluster can alter the value of the cluster centroid by up to  $\Delta(X)/k$ , where  $\Delta(X)$  is the maximum distance between elements in the domain of the input data (we are assuming that centroids are computed as the arithmetic average of record values in the cluster).

The above worst-case scenario overestimates the actual sensitivity of the output and, thus, the noise to be added to the centroids to achieve  $\varepsilon$ -differential privacy. Indeed, it is highly unlikely that modifying one input record by up to  $\Delta(X)$  would change by  $\Delta(X)$  one record in *each* cluster. Let us consider an extreme scenario, in which all records in the input data set take the *maximum* possible value tuple in the domain of  $X$ . Recall that

the insensitive microaggregation used sorts and groups records according to a total order defined over the domain of  $X$ . Then, assume that the record located in the last position of the sorted list changes to take the *minimum* value tuple of the domain of  $X$ , so that its distance to any of the other records in the data set is  $\Delta(X)$ . According to the ordering criterion, such a change would cause the modified record to be “inserted” in the first position of the sorted list. Consequently, all other records would be moved to the next position, which would change *all* clusters by one record. However, from the perspective of the centroid computation (*i.e* the average of the record in the group), only the first cluster centroid, where the modified record is located, would change and its variation would be exactly  $\Delta(X)/k$ .

In other intermediate scenarios, the effect of modifying one record would be split among the centroids of the clusters affected by the modification. Intuitively, the aggregation of the centroid variations would seem to be upper-bounded by  $\Delta(X)/k$ , which corresponds to the extreme case detailed above. However, this is only true if a total order for the domain of  $X$  exists for which the triangular inequality is satisfied, that is, when  $d(r_1, r_2) + d(r_2, r_3) \geq d(r_1, r_3)$  holds for any records  $r_1$ ,  $r_2$  and  $r_3$  in  $X$ . Unfortunately, this is generally not the case for multivariate data because a natural total order does not always exist. Artificial total orders defined for multivariate data (for example, see the proposal in [17]) do not fulfill the triangular inequality and, as discussed above, the sensitivity of individual centroids should be multiplied by the number of released centroids ( $n/k \times \Delta(X)/k$ ) to satisfy differential privacy.

The lack of a total order does not occur in univariate numerical data sets, that is, those with just one attribute. With a single numerical attribute, a natural total order (the usual numerical order) can be easily defined with respect to the minimum or maximum value of the domain of values of the attribute so that the triangular inequality is fulfilled. In these conditions, it is shown in [9] that clusters in the optimal microaggregation partition contain consecutive values. The next lemma shows that the sensitivity of the set of centroids is indeed  $\Delta(X)/k$ .

**Lemma 1.** *Let  $x_1, \dots, x_n$  be a totally ordered set of values that has been microaggregated into  $\lfloor n/k \rfloor$  clusters of  $k$  consecutive values each, except perhaps one cluster that contains up to  $2k - 1$  consecutive values. Let the centroids of these clusters be  $\bar{x}_1, \dots, \bar{x}_{\lfloor n/k \rfloor}$ , respectively. Now if, for any single  $i$ ,  $x_i$  is replaced by  $x'_i$  such that  $|x'_i - x_i| \leq \Delta$  and new clusters and*

centroids  $\bar{x}'_1, \dots, \bar{x}'_{\lfloor n/k \rfloor}$  are computed, it holds that

$$\sum_{j=1}^{\lfloor n/k \rfloor} |\bar{x}'_j - \bar{x}_j| \leq \Delta/k.$$

**Proof.** Assume without loss of generality that  $x'_i > x_i$  (the proof for  $x'_i < x_i$  is symmetric). Assume, for the sake of simplicity, that  $n$  is a multiple of  $k$  (we will later relax this assumption). Hence, exactly  $n/k$  clusters are obtained, with cluster  $j$  containing consecutive values from  $x_{(j-1)k+1}$  to  $x_{jk}$ . Let  $j_i$  be the cluster to which  $x_i$  belongs. We can distinguish two cases, namely  $x'_i \leq x_{j_i k+1}$  and  $x'_i > x_{j_i k+1}$ .

*Case 1.* When  $x'_i \leq x_{j_i k+1}$ ,  $x'_i$  stays in  $j_i$ . Thus, the centroids of all clusters other than  $j_i$  stay unchanged and the centroid of cluster  $j_i$  increases by  $\Delta/k$ , because  $x'_i = x_i + \Delta$ . So the lemma follows in this case.

*Case 2.* When  $x'_i > x_{j_i k+1}$ , two or more clusters change as a result of replacing  $x_i$  by  $x'_i$ : cluster  $j_i$  loses  $x_i$  and another cluster  $j'_i$  (for  $j'_i > j_i$ ) acquires  $x'_i$ . To maintain its cardinality  $k$ , after losing  $x_i$ , cluster  $j_i$  acquires  $x_{j_i k+1}$ . In turn, cluster  $j_i + 1$  loses  $x_{j_i k+1}$  and acquires  $x_{(j_i+1)k+1}$ , and so on, until cluster  $j'_i$ , which transfers its smallest value  $x_{(j'_i-1)k+1}$  to cluster  $j'_i - 1$  and acquires  $x'_i$ . From cluster  $j'_i + 1$  upwards, nothing changes. Hence the overall impact on centroids is

$$\begin{aligned} \sum_{j=1}^{n/k} |\bar{x}'_j - \bar{x}_j| &= \sum_{j=j_i}^{j'_i} |\bar{x}'_j - \bar{x}_j| \\ &= \frac{x_{j_i k+1} - x_i}{k} + \frac{x_{(j_i+1)k+1} - x_{j_i k+1}}{k} + \dots + \frac{x'_i - x_{(j'_i-1)k+1}}{k} \\ &= \frac{x'_i - x_i}{k} = \frac{\Delta}{k}. \end{aligned} \tag{2}$$

Hence, the lemma follows also in this case.

Now consider the general situation in which  $n$  is not a multiple of  $k$ . In this situation there are  $\lfloor n/k \rfloor$  clusters and one of them contains between  $k+1$  and  $2k-1$  values. If we are in Case 1 above and this larger cluster is cluster  $j_i$ , the centroid of  $j_i$  changes by less than  $\Delta/k$ , so the lemma also holds; of course if the larger cluster is one of the other clusters, it is unaffected and the lemma also holds. If we are in Case 2 above and the larger cluster is one of the clusters that change, one of the fractions in the third term of Expression (2) above has denominator greater than  $k$  and hence the overall sum is less than  $\Delta/k$ , so the lemma also holds; if the larger cluster is one of the unaffected ones, the lemma also holds.  $\square$

### 3.2. Sensitivity reduction in multivariate data sets via individual ranking microaggregation

From the previous section, it turns out that, for univariate data sets, the amount of noise needed to fulfill differential privacy after the microaggregation step is significantly lower than with the method in [17] (*i.e.*  $\Delta(X)/k$  vs.  $n/k \times \Delta(X)/k$ ). Moreover, this noise is exactly  $1/k$ -th of the noise required by the standard differential privacy approach, in which the sensitivity is  $\Delta(X)$  because *any* output record may change by  $\Delta(X)$  following a modification of any record in the input (as also stated in [42] when sorting attributes by their value counts).

To benefit from such a noise reduction in the case of multivariate data sets, we rely on the following composition property of differential privacy.

**Lemma 2 (Sequential composition [43]).** *Let each sanitizing algorithm  $Ag_i$  in a set of sanitizers provide  $\varepsilon_i$ -differential privacy. Then a sequence of sanitizers  $Ag_i$  applied to a data set  $D$  provides  $(\sum_i \varepsilon_i)$ -differential privacy.*

In the context of differentially private data publishing, we can think of a data release as the collected answers to successive queries for each record in the data set. Let  $I_r(X)$  be the query that returns the value of record  $r$  (from a total of  $n$  records) in the data set  $X$ . In turn, we can think of  $I_r(X)$  as the collected answers to successive queries for each of the attributes of record  $r$ . Let  $I_{ra}(X)$  be the query function that returns the value of attribute  $a$  (from a total of  $m$  attributes). We have  $I_r(X) = (I_{r1}(X), \dots, I_{rm}(X))$ . The differentially private data set that we seek can be generated by giving a differentially private answer to the set of queries  $I_{ra}(X)$  for all  $r = 1, \dots, n$  and all  $a = 1, \dots, m$ . Differential privacy being designed to protect the privacy of individuals, such queries are very sensitive and require a large amount of noise.

To reduce sensitivity and hence the amount of noise needed to attain differential privacy, we rely on individual ranking microaggregation (which is more utility-preserving than multivariate microaggregation, as explained in Section 2.1 above). Instead of asking for  $I_{ra}(X)$ , the data set is generated by asking for individual ranking microaggregation centroids. Let  $\rho_X(r, a)$  be the group of records of data set  $X$  in the individual ranking microaggregation of attribute  $a$  that corresponds to  $r$ , and let  $C_{\rho_X(r, a)}$  be associated centroid. We replace  $I_{ra}(X)$  by  $C_{\rho_X(r, a)}$ .

Now, we have to minimize the amount of noise required to answer these queries in a differentially private manner. We work with each attribute independently and then combine the queries corresponding to different attributes

by applying sequential composition. If we get an  $\varepsilon$ -differentially private response to  $(C_{\rho_X(1,a)}, \dots, C_{\rho_X(n,a)})$  for each  $a = 1, \dots, m$ , then joining them we have  $m\varepsilon$ -differential privacy.

For attribute  $a$ , we have to answer the query  $(C_{\rho_X(1,a)}, \dots, C_{\rho_X(n,a)})$  in an  $\varepsilon$ -differentially private manner. If we compute the  $L_1$ -sensitivity of this query,  $s_a$ , we can attain  $\varepsilon$ -differential privacy by adding a Laplace distributed noise with scale parameter  $s_a/\varepsilon$  to each component  $C_{\rho_X(i,a)}$ . We have already seen that for individual ranking microaggregation the  $L_1$ -sensitivity of the list of centroids is  $\Delta_a/k$ . However, in our query each centroid appears  $k$  (or more times); hence, the sensitivity is multiplied by  $k$  and becomes  $\Delta_a$  (or greater), which is not satisfactory. Our goal is to show that we can attain  $\varepsilon$ -differential privacy by adding a Laplace noise with scale  $\Delta_a/(k\varepsilon)$  rather than  $\Delta_a/\varepsilon$  (as an  $L_1$ -sensitivity of  $\Delta_a$  would require). To that end, instead of taking an independent draw of the noise distribution for each of the components, we use the same draw for all the components that refer to the same centroid. That is, we use the same random variable  $L_{\rho_X(r,a)}$  to determine the amount of noise that is added to all the components sharing the same value  $C_{\rho_X(r,a)}$ ; similarly, in data set  $X'$  we use  $L_{\rho_{X'}(r,a)}$  as noise for all components sharing the same value  $C_{\rho_{X'}(r,a)}$ . For  $\varepsilon$ -differential privacy, it must be

$$\frac{\Pr((C_{\rho_X(1,a)} + L_{\rho_X(1,a)}, \dots, C_{\rho_X(n,a)} + L_{\rho_X(n,a)}) = (x_1, \dots, x_n))}{\Pr((C_{\rho_{X'}(1,a)} + L_{\rho_{X'}(1,a)}, \dots, C_{\rho_{X'}(n,a)} + L_{\rho_{X'}(n,a)}) = (x_1, \dots, x_n))} \leq \exp(\varepsilon).$$

If any of  $x_1, \dots, x_n$  is not a centroid value plus the noise corresponding to that centroid value (note that equal centroid values are added equal noise values, as said above), the probabilities in both the numerator and the denominator of the above expression are zero, and differential privacy is satisfied. Otherwise, we have that  $x_1, \dots, x_n$  are only repetitions of  $n/k$  different values, that is, the values of the  $n/k$  centroids plus the noise corresponding to each centroid value. Thus, we can simplify the expression by removing all but one of each of those repetitions. Let  $C_{i,a}(X)$  and  $C_{i,a}(X')$  for  $i = 1, \dots, n/k$  be the centroid values for attribute  $a$  associated to  $X$  and  $X'$ , respectively, and  $L_{i,a}$  and  $L'_{i,a}$  be Laplace noises with scale  $\Delta_a/(k\varepsilon)$  associated to those centroid values, respectively. After rewriting the above inequality in these terms and taking into account that the sensitivity of the list of centroids is  $\Delta_a/k$ , it is evident that  $\varepsilon$ -differential privacy is satisfied.

$$\frac{\Pr((C_{1,a}(X) + L_{1,a}, \dots, C_{n/k,a}(X) + L_{n/k,a}) = (x'_1, \dots, x'_{n/k}))}{\Pr((C_{1,a}(X') + L'_{1,a}, \dots, C_{n/k,a}(X') + L'_{n/k,a}) = (x'_1, \dots, x'_{n/k}))} \leq \exp(\varepsilon).$$

Hence, we propose the following algorithm to obtain a differentially private version  $X^D$  of a numerical original data set  $X$  with attributes  $A_1, \dots, A_m$ .

**Algorithm 1.**

1. Use individual-ranking microaggregation independently on each attribute  $A_i$ , for  $i = 1$  to  $m$ . Within each cluster, all attribute values are replaced by the cluster centroid value, so each microaggregated cluster consists of  $k$  repeated centroid values. Let the resulting microaggregated data set be  $X^M$ .
2. Add Laplace noise independently to each attribute  $A_i^M$  of  $X^M$ , where the scale parameter for attribute  $A_i^M$  is

$$\Delta(A_i^M)/\varepsilon = \Delta(A_i)/(k \times \varepsilon).$$

The same noise perturbation is used on all repeated centroid values within each cluster.

Now we can state:

**Lemma 3.** *The data set output by Algorithm 1 is  $m\varepsilon$ -differentially private.*

**Proof.** The lemma follows from the discussion preceding Algorithm 1.

□

**Note.** In Step 2 of Algorithm 1, it is critically important to add exactly the same noise perturbation to all repeated values within a microaggregated cluster. If we used different random perturbations for each repeated value, the resulting noise-added cluster would be equivalent to the answers to  $k$  independent queries. This would multiply by  $k$  the sensitivity of the centroid, which would cancel the sensitivity reduction brought by microaggregation in Step 1.

### 3.3. Choosing the microaggregation parameter $k$

In order to obtain an  $\varepsilon$ -differentially private data set, by parallel composition it suffices to make each record  $\varepsilon$ -differentially private. In turn, to make a record  $\varepsilon$ -differentially private, we have two possibilities:

1. *Plain Laplace noise addition without microaggregation.* Given that each record has  $m$  attributes, by sequential composition we need  $(\varepsilon/m)$ -differentially private attribute values to obtain an  $\varepsilon$ -differentially private record. Hence, Laplace noise addition with scale parameter  $\Delta(A_i)/(\varepsilon/m) = m\Delta(A_i)/\varepsilon$  needs to be added to each attribute  $A_i$ .

2. *Our approach.* When performing individual-ranking microaggregation and replacing original values by cluster centroids, we preserve the structure of records. By sequential composition, to make a record of  $X^M$   $\varepsilon$ -differentially private, we need to make attributes in  $X^M$   $(\varepsilon/m)$ -differentially private. Hence, Laplace noise addition with scale parameter  $\Delta(A_i^M)/(\varepsilon/m) = m\Delta(A_i^M)/\varepsilon$  needs to be added to each attribute  $A_i^M$ . However, dealing with  $A_i^M$  rather than  $A_i$  is better, because  $A_i^M$  is less sensitive. Indeed,  $\Delta(A_i^M) = \Delta(A_i)/k$ , so the scale parameter is  $m\Delta(A_i^M)/(k\varepsilon)$ .

According to the above discussion, our approach adds less noise than plain Laplace noise addition for any  $k > 1$ . Admittedly, its prior individual ranking microaggregation causes some additional information loss. However, this information loss grows very slowly with the cluster size  $k$  and also with the number of attributes  $m$  (see [20]), whereas the Laplace noise being added decreases very quickly with  $k$ . The experiments in Section 4 below show that the information loss caused by individual ranking is negligible in front of Laplace noise addition.

#### 3.4. Dealing with categorical attributes

So far, we have assumed that attributes in the input data set are numerical, so that: i) they are totally ordered; ii) centroids can be computed as standard numerical averages; and iii) Laplace noise with an appropriate scale parameter can be added to satisfy differential privacy. However, many data sets contain attributes with categorical values, such as Ethnicity, Country of birth or Job. Unlike numerical attributes, categorical attributes take values from a finite set of categories for which the arithmetical operations needed to microaggregate and add noise to the outputs do not make sense. Following the discussions in [17], some alternative mechanisms can be used to adapt the above-described method to categorical attributes:

- Unlike numbers, the domain of values of a categorical attribute should be defined by extension. Ways to do this are a flat list or a hierarchy/taxonomy. The latter is more desirable, since the taxonomy implicitly captures the semantics inherent to categorical values (*e.g.*, disease categories, job categories, sports categories, etc.). In this manner, further operations can exploit this taxonomic knowledge and provide a semantically coherent management of attribute values, which is usually the most important dimension of utility for categorical data [26].
- A suitable function is needed to compare categorical values that exploits the semantics provided by the corresponding taxonomies (if

any). Semantic distance measures quantify the amount of differences observed between two categorical values according to the knowledge modeled in a taxonomy. In [17] several measures available in the literature were discussed from the perspective of differential privacy, and the measure proposed in [44] was evaluated as the most suitable one. It computes the distance  $\delta : A_i \times A_i \rightarrow \mathbb{R}$  between two categorical values  $a_1^i$  and  $a_2^i$  of attribute  $A_i$ , whose domain is modeled in the taxonomy  $\tau(A_i)$ , as a logarithmic function of their number of non-common taxonomic ancestors divided (for normalization) by their total number of ancestors:

$$\delta(a_1^i, a_2^i) = \log_2 \left( 1 + \frac{|\phi(a_1^i) \cup \phi(a_2^i)| - |\phi(a_1^i) \cap \phi(a_2^i)|}{|\phi(a_1^i) \cup \phi(a_2^i)|} \right), \quad (3)$$

where  $\phi(a_j^i)$  is the set of taxonomic ancestors of  $a_j^i$  in  $\tau(A_i)$ , including itself.

The advantages of this measure are: i) it captures subtle differences between values modeled in the taxonomy because all taxonomic ancestors are considered; ii) thanks to its non-linearity, its sensitivity to outlying values is low, which is desirable to reduce the sensitivity of data; iii) it fulfills the following measure properties: *non-negativity*, *reflexivity*, *symmetry* and *triangle inequality*, thereby defining a coherent total order within the attribute categories.

- A total order that yields insensitive and within-cluster homogeneous microaggregation can be defined through the notion of *marginality* [45]. Marginality measures how far each categorical value of the attribute in the data set lies from the “center” of that attribute’s taxonomy, according to a semantic distance (like the above-described one). A total order between categorical values can be defined based on their marginality: categorical values present in the data set are sorted according to their distance to the most marginal value (*i.e.* the categorical value farthest from the center of the domain). The *marginality*  $m(\cdot, \cdot)$  of each value  $a_j^i$  in  $A_i$  with respect to its domain of values  $Dom(A_i)$  is computed as

$$m(Dom(A_i), a_j^i) = \sum_{a_l^i \in Dom(A_i) - \{a_j^i\}} \delta(a_l^i, a_j^i) \quad (4)$$

where  $\delta(\cdot, \cdot)$  is the semantic distance between two values. The greater  $m(Dom(A_i), a_j^i)$ , the more marginal (*i.e.*, the less central) is  $a_j^i$  with regard to  $Dom(A_i)$ .

- Microaggregation replaces original values in a cluster by the cluster centroid, which is the arithmetical mean in case of numerical data. Centroids for categorical data can be obtained by relying again on marginality: the mean of a sample of categorical values can be approximated by the least marginal value in the taxonomy, that is, the value that minimizes the aggregated distances to all other elements in the data set [46]. Formally, given a sample  $S(A_i)$  of a nominal attribute  $A_i$  in a certain cluster, the marginality-based centroid for that cluster is defined as

$$\text{Centroid}(S(A_i)) = \arg \min_{a_j^i \in \tau(S(A_i))} m(S(A_i), a_j^i), \quad (5)$$

where  $\tau(S(A_i))$  is the minimum taxonomy extracted from  $\tau(A_i)$  that includes all values in  $S(A_i)$ .

- Finally, to satisfy differential privacy, an amount of uncertainty proportional to each attribute’s sensitivity should be added prior to releasing the data. Since adding Laplace noise to categorical centroids makes no sense, an alternative way to obtain differentially private outputs consists in selecting attribute centroids in a probabilistic manner. This can be done by means of the Exponential Mechanism proposed in [47]. This mechanism chooses the centroid closest to the optimum (*i.e.* least marginal value, in our case) according to the input data, the  $\varepsilon$ -differential privacy parameter and a quality criterion, which in this case is the marginality of each categorical value. Formally, given a function with discrete outputs  $t$ , the mechanism chooses an output that is close to the optimum according to the input data  $X$  and quality criterion  $q(X, t)$ , while preserving  $\varepsilon$ -differential privacy. Each output is associated with a selection probability  $\text{Pr}(t)$ , which grows exponentially with the quality criterion, as follows:

$$\text{Pr}(t) \propto \exp\left(\frac{\varepsilon q(X, t)}{2\Delta(q)}\right)$$

Algorithm 2 describes the application of this mechanism to select  $\varepsilon$ -differentially private centroids.

**Algorithm 2.**

*let  $C$  be a cluster with at least  $k$  values of the attribute  $A_i$*

1. Take as the quality criterion  $q(\cdot, \cdot)$  for each centroid candidate  $a_j^i$  in  $\tau(A_i)$  the additive inverse of its marginality towards the attribute values  $S(A_i)$  contained in  $C$ , that is,  $-m(S(A_i), a_j^i)$ ;
2. Sample the centroid from a distribution that assigns

$$\Pr(a_j^i) \propto \exp\left(\frac{\varepsilon \times (-m(S(A_i), a_j^i))}{2\Delta(m(A_i))}\right). \quad (6)$$

#### 4. Empirical evaluation

This section details the empirical evaluation of the proposed method (in terms of noise reduction and utility preservation) in comparison with the method in [17].

##### 4.1. Evaluation data

As evaluation data we used two data sets:

- “Census” [48], which is a standard data set meant for testing privacy protection methods. It was used in the European project CASC and in [49, 50, 51, 10, 25] and it contains 1,080 records with 13 numerical attributes. Since all attributes represent non-negative numerical magnitudes (*i.e.* money amounts), we defined the domains of the attributes as  $[0 \dots (1.5 \times \text{max\_attr\_value\_in\_dataset})]$ . The domain upper bound is a reasonable estimate if the attribute values in the data set are representative of the attribute values in the population, which in particular means that the population outliers are represented in the data set. The difference between the bounds of the domain of each attribute  $A_i$  determines the sensitivity of that attribute ( $\Delta(A_i)$ ) and, as detailed above, determines the amount of Laplace noise to be added to microaggregated outputs. Since the Laplace distribution takes values in the range  $(-\infty, +\infty)$ , for consistency we bound noise-added outputs to the domain ranges define above.
- “Adult” [52], which is a well-known data set from the UCI repository. It contains both numeric and categorical attributes and it was also used in [17]. To enable a fair comparison, we used the same attributes as in [17]. As categorical attributes, we used OCCUPATION (that covers 14 distinct categories) and NATIVE-COUNTRY (with 41 categories). The taxonomies modeling attribute domains were extracted from WordNet 2.1 [53], a general-purpose repository that taxonomically models more than 100,000 concepts. Mappings between attribute

labels and WordNet concepts are those stated in [26]. Domain boundaries for each attribute and sensitivities for centroid quality criteria were set as described in Section 3.4. As numeric attributes, we used AGE and (working) HOURS-PER-WEEK. Domain boundaries and sensitivities for these two numerical attributes were computed as explained for “Census”. The experiments were carried out with the training corpus from the Adult data set, which consists of 30,162 records after removing records with missing values of the considered attributes.

#### 4.2. Evaluation measures and experiments

Since our proposal aims at providing differentially private outputs while making as few assumptions on their uses as  $k$ -anonymity-like models do, we used generic utility metrics, as usual in the literature on  $k$ -anonymity and statistical disclosure control (*e.g.* [10]). In that literature, the utility of the anonymized output is evaluated in terms of *information loss*. Information loss measures the differences between the original and the anonymized data set.

A well-known generic information loss measure that is suitable to capture the distortion of the output is the Relative Error (RE), which measures the error of an answer as a function of the magnitude of such answer; in this manner, answers with large magnitudes can tolerate larger errors [54]. In our setting, for each numerical attribute value  $a_j^i$ , the relative error is measured as the absolute difference between each original attribute value  $a_j^i$  and its masked version  $(a_j^i)'$  divided by the original value.

$$Relative\_Error(a_j^i) = \frac{|a_j^i - (a_j^i)'|}{\max(sanity\_bound(A_i), |a_j^i|)}$$

A sanity bound is included in the denominator to mitigate the effect excessively small values. Because in our approach we aim at publishing the attribute values of the records in the data set, we defined this sanity bound as a function of the domain of values each attribute; specifically,  $sanity\_bound(A_i) = Dom(A_i)/100$ . In this manner, the mitigation effect of the sanity bound is adapted to the order of magnitude of each attribute.

For categorical attributes we directly measured the error as the semantic distance (Expression (3)) between original and masked records, which is already normalized in the 0..1 range. The Relative Error of the whole data set  $X$  is measured as the average Relative Error of all attributes of all records. Notice that with a high error, that is, a high information loss, a

lot of data uses are severely damaged, like for example subdomain analyses (analyses restricted to parts of the data set).

Another generic information loss measure focuses on how different are variances of attributes in the original and anonymized data sets [55]. Preserving the sample variance is relevant for statistical analysis and closely depends on how constrained the microaggregation algorithm is. The variation of the attribute variances (for numerical attributes) has been measured as follows:

$$\Delta(\sigma^2(A_i)) = \frac{|\sigma^2(A'_i) - \sigma^2(A_i)|}{|\sigma^2(A_i)|},$$

where  $\sigma^2(A_i)$  and  $\sigma^2(A'_i)$  denote the variances of attribute  $A_i$  in the original data set and its masked version, respectively.

Moreover, since many works on differential privacy focus on preserving the utility of counting queries [35, 31, 32, 27, 28, 29, 30], we measured how the methods preserve the data distribution by building histograms of each attribute and comparing the distribution between the original and masked values according to the well-known Jensen-Shannon divergence (JSD) [56], which is symmetric and bounded in the 0..1 range. At a data set level, we averaged the divergence of all the attributes. Histograms for continuous attributes (i.e., those of the “Census” data set) have been created by grouping records in bins accounting 1/100th of the attribute domain. In the “Adult” data set, in which all the attributes (either numeric or categorical) are discrete and have a limited set of values, each bin corresponds to one individual attribute value.

The  $\varepsilon$  parameter for differential privacy was set to  $\varepsilon = \{0.1, 1, 10\}$ , which covers the usual range of differential privacy levels observed in the literature [57, 58, 59, 33, 17]. As discussed in Section 3.2, the scale parameter of the Laplace noise needed by our method to achieve  $\varepsilon$ -differential privacy is  $\Delta(A_i)/(k \times (\varepsilon/m))$ ; that is, it depends on the level  $k$  of prior microaggregation and on the number  $m$  of attributes to be protected. We evaluated the influence of these parameters in the “Census” data set by taking  $k$  between 2 and 100 and  $m \in \{1, 8, 13\}$ . For the “Adult” data set, as done in [17], we set  $k$  between 2 and 200.

#### 4.3. Comparison with baseline methods

In order to benchmark the results of our proposal, we considered the following baseline methods:

- Plain Laplace noise addition for  $\varepsilon$ -differential privacy as described in Section 3.3 above. Even though this mechanism is the naivest way to

produce differentially private data sets, it is useful as an upper bound of information loss. By comparing against it, we can quantify the gain brought by the prior microaggregation steps.

- Plain individual ranking, with no subsequent Laplace noise addition. Although this method does not lead to  $\varepsilon$ -differential privacy by itself, we want to show the contribution of individual ranking to the information loss caused by our method.

We computed RE and JSD for the baseline methods above and our approach for the two evaluation data sets. Figure 1 shows the comparison between plain Laplace noise addition, plain individual ranking and our approach for “Census”; Figure 2 shows the same comparison for “Adult”. Due to the broad ranges of the RE values, a  $\log_{10}$  scale is used for the Y-axes.

The plain Laplace noise addition baselines are displayed as gray horizontal lines, because they do not depend on the value of  $k$ . Each test involving Laplace noise shows the average results of 10 runs, for the sake of stability. In any case, the spikes shown in the graphs are the result of the randomization inherent to Laplace noise addition.

Both Figure 1 and Figure 2 show that, already for  $k > 1$ , our approach reduces the noise required to attain  $\varepsilon$ -differential privacy compared to plain Laplace noise addition. This confirms what was said in Section 3.3.

The relative improvement of RE depends on the value of  $\varepsilon$ . For  $\varepsilon = 0.1$  the amount of noise involved is so high that even with the noise reduction achieved by our method, the output data are hardly useful. For  $\varepsilon = 10$  there is a substantial decline of RE for low  $k$  and, for larger  $k$ , RE stays nearly constant and is almost as low as the RE achieved by individual ranking alone. This is especially noticeable when only one attribute is protected (for the “Census” data set) for which the information loss of the proposed method even grows for  $k > 25$ ; with a single attribute, we do not need to apply sequential composition (that would require more noise to be added) in order to attain a certain level of  $\varepsilon$ -differential privacy. In this case, the distortion added by individual ranking in larger clusters limits the effect of the noise reduction achieved at the  $\varepsilon$ -differential privacy stage due to the decreased sensitivity with larger  $k$ . For the “Adult” data set, the difference in RE between the  $\varepsilon$ -differentially private outcome and the plain individual ranking is more noticeable because of the need to discretize noise-added values. However, the larger cardinality of the data set also allows using larger  $k$  values (to reduce the sensitivity) than in “Census” with comparable utility damage.

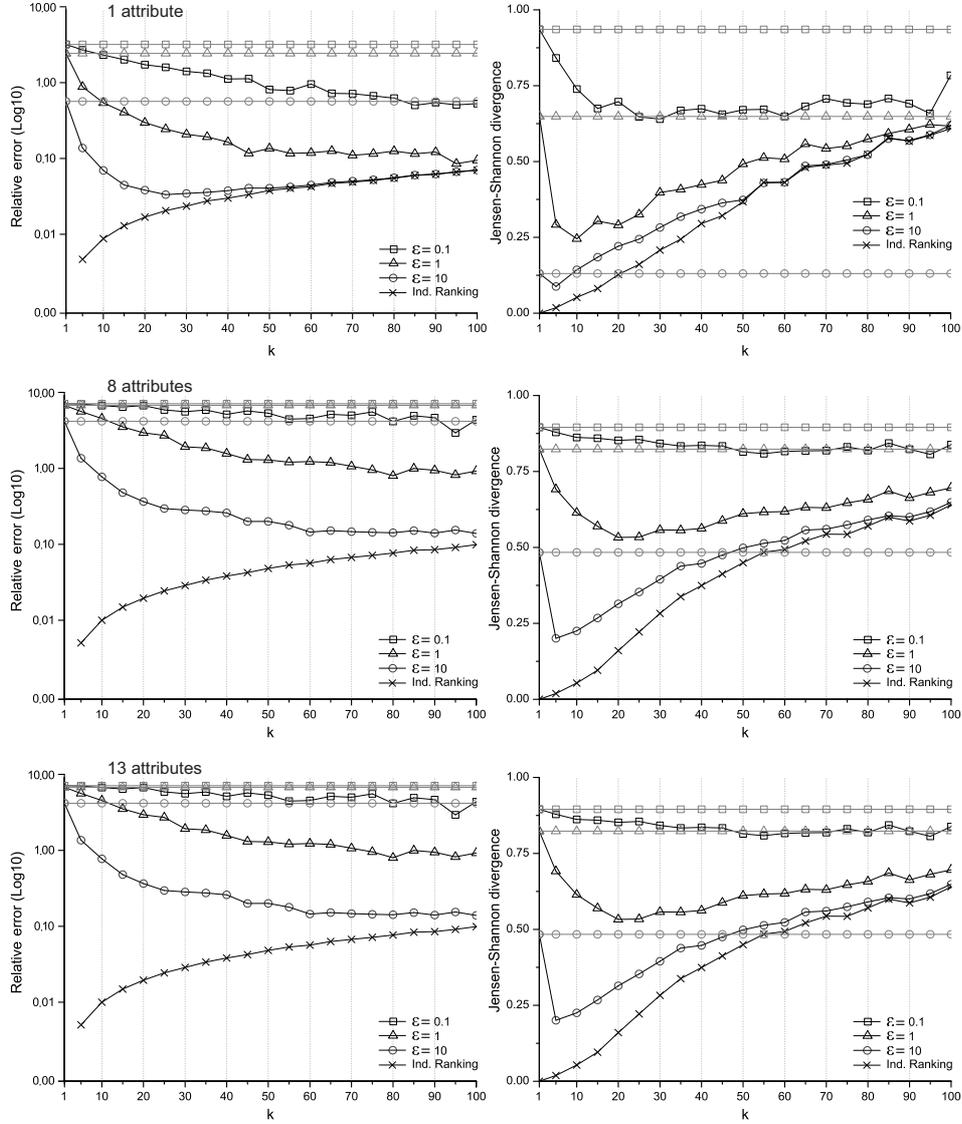


Figure 1: “Census” data set: RE (on the left) and JSD (on the right) for the proposed method for several numbers of attributes ( $m$ ) and  $\epsilon$  values (black non-horizontal lines, as RE and JSD depend on the microaggregation parameter  $k$ ) compared to plain Laplace noise addition (gray horizontal lines, because RE and JSD do not depend on  $k$ ) and plain individual ranking microaggregation.

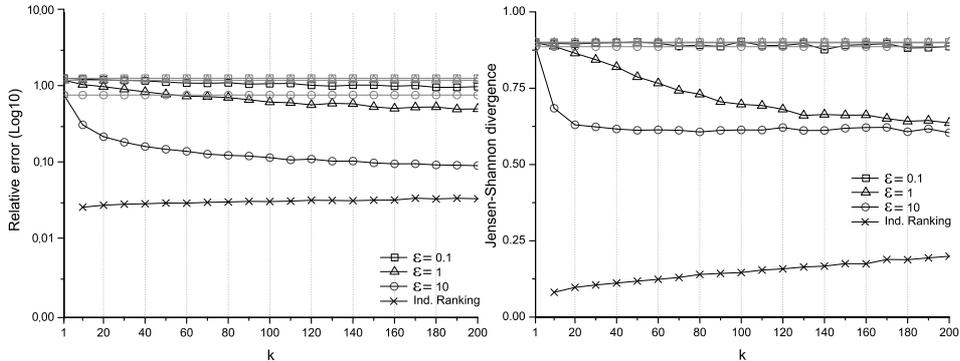


Figure 2: “Adult” data set: RE (on the left) and JSD (on the right) for the proposed method for several  $\varepsilon$  values (black non-horizontal lines, as RE and JSD depend on the microaggregation parameter  $k$ ) compared to plain Laplace noise addition (gray horizontal lines, because RE and JSD do not depend on  $k$ ) and plain individual ranking microaggregation.

The comparison of distributions between the original and the masked data also outputs a similar pattern: JSD improves for  $k > 1$ . For the “Adult” data set, Figure 2 shows that the behaviors of JSD and RE are similar. For the “Census” data set, however, Figure 1 reports a sharp decline of JSD for low  $k$ , whereas for large  $k$  the divergence of distributions tends to increase; this is specially noticeable for  $\varepsilon = 10$ , whose results for  $k > 10$  (for a single attribute) and for  $k > 45$  (for multiple attributes) are even worse than with plain Laplace noise. In these cases, the distortion introduced by the individual ranking microaggregation dominates the gain brought by the reduced noise addition. This is clearer when the number of attributes is small, because so is the noise to be added to fulfill  $\varepsilon$ -differential privacy. It is important to note that, for the “Census” data set, the continuous values of the attributes have been discretized in bins covering 1/100th of the attribute domain. Thus, a microaggregation with a low  $k$  would tend to cluster values that fall within the same bin, which explains the low distortion incurred in the (discretized) distributions. For larger  $k$ , however, microaggregation tends to group records of different bins, which significantly alters the data distribution; this is precisely what happens for the “Adult” data set for any value of  $k$ , because bins cover individual attribute values.

#### 4.4. Comparison with prior multivariate microaggregation

In a second battery of experiments, we compared the proposed method with the previous work [17], in which records were microaggregated using

an insensitive version of the MDAV multivariate algorithm [10].

Similarly to previous figures, Figures 3 and 4 depict the RE and JSD values for the different parameterizations of  $k$ ,  $\varepsilon$  and  $m$ , for the “Census” and “Adult” data sets, respectively. The results of the proposed method are represented with black lines whereas the results of the previous work [17] are displayed by gray lines with the same pattern for each  $\varepsilon$  value. As baselines, we also added the RE and JSD values incurred by the individual ranking and the insensitive MDAV microaggregation algorithms alone.

First, we notice that insensitive MDAV multivariate microaggregation alone incurs a noticeably higher RE than plain individual ranking microaggregation for  $m > 1$ . JSD is also higher for “Census” and significantly higher for “Adult”, because in the latter case the already discrete values are not grouped into bins and, thus, any change is reflected as a divergence in distributions. In fact, for “Adult” our differentially private method is even able to improve the figures of insensitive MDAV for  $k > 100$ . For  $m = 1$ , both types of microaggregation are equivalent, so their REs and JSDs overlap. On the other hand, as discussed in Section 3.3, for multivariate data sets ( $m > 1$ ), individual ranking microaggregation yields more homogeneous clusters than MDAV multivariate microaggregation, because the former method builds clusters on an attribute-by-attribute basis, whereas the latter builds clusters by taking all attributes together. Moreover, the insensitive version of MDAV required for the method in [17] to fulfill differential privacy produces yet less homogeneous clusters, due to the artificial total order enforced for input records.

When comparing both differentially private approaches, our current proposal offers a significant improvement in most cases (for RE and JSD), but such an improvement depends on the number of attributes to be protected. For  $m = 1$ , we observe the largest differences between both methods, because the scale parameter of our approach is just  $\Delta(A)/(k \times \varepsilon)$  whereas it is  $n/k \times (\Delta(A)/(k \times \varepsilon))$  for [17]. In fact, even for the smallest  $\varepsilon$  value ( $\varepsilon = 0.1$ ), the reduction of information loss is noticeable. However, as shown also in the previous experiments, for the largest  $\varepsilon$  value ( $\varepsilon = 10$ ), the distortion caused by individual ranking dominates the small amount noise subsequently added to satisfy differential privacy, which is especially noticeable for the JSD; for  $k > 20$ (for RE) or  $k > 10$  (for JSD) this effect overrides the improvement brought by our proposed method.

When the number  $m$  of attributes to be protected increases, the effective reduction of RE and JSD achieved by our approach decreases, because the scale of the required noise increases as  $\Delta(A)/(k \times (\varepsilon/m))$ , whereas it stays constant for the method in [17]. Indeed, using prior individual-ranking

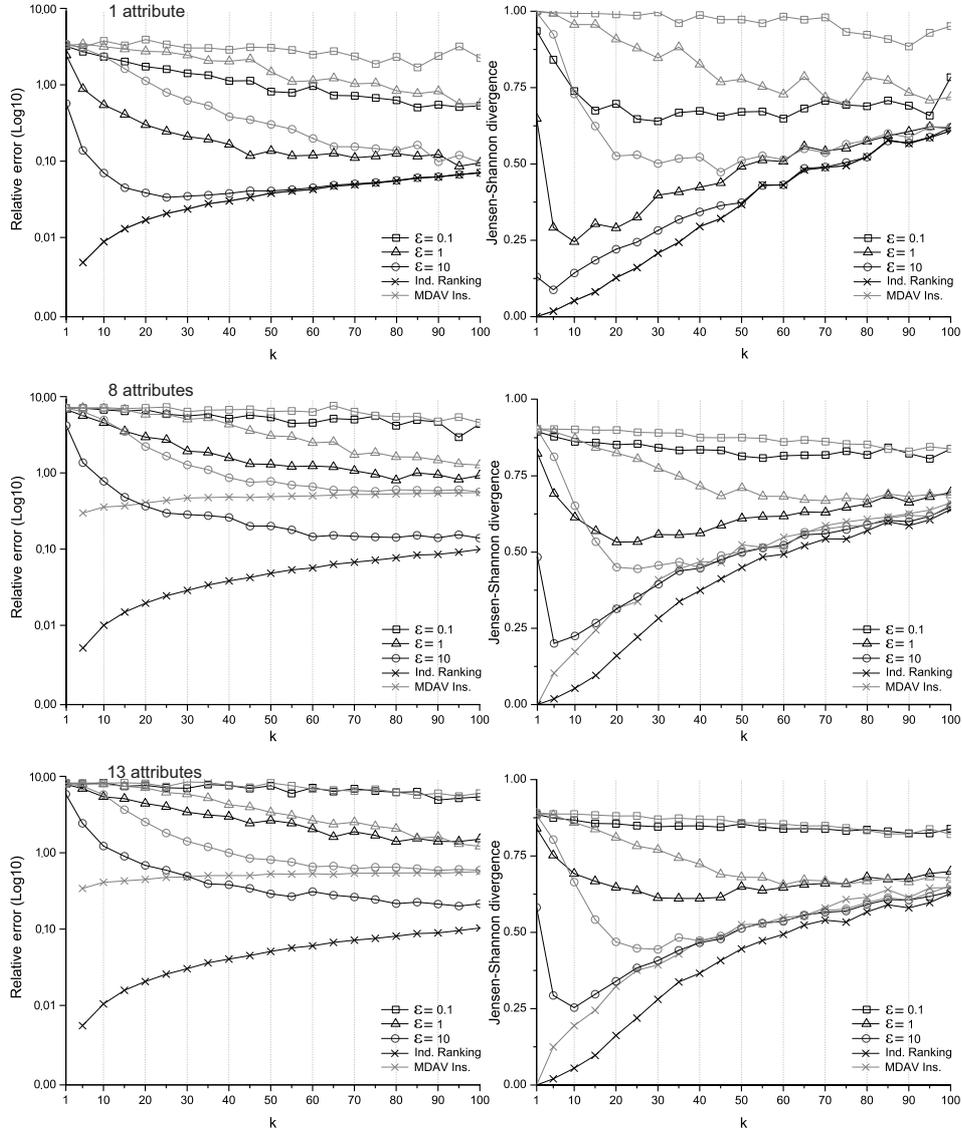


Figure 3: “Census” data set: RE (on the left) and JSD (on the right) in the proposed method for several numbers of attributes ( $m$ ) and  $\epsilon$  values (black dashed lines) compared to the previous work [17] (gray dashed lines), the insensitive version of the MDAV microaggregation algorithm (gray solid lines) and the plain individual ranking method (black solid lines).

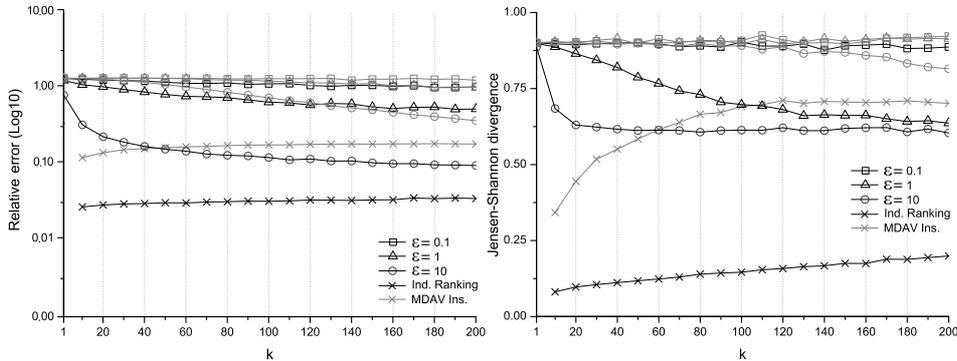


Figure 4: “Adult” data set: RE (on the left) and JSD (on the right) in the proposed method for several  $\varepsilon$  values (black dashed lines) compared to the previous work [17] (gray dashed lines), the insensitive version of the MDAV microaggregation algorithm (gray solid lines) and the plain individual ranking method (black solid lines).

microaggregation requires subsequently adding noise whose scale factor increases with  $m$  in order to attain  $\varepsilon$ -differential privacy, whereas the noise to be added after multivariate microaggregation does not depend on  $m$  (see Section 3.2 above). When  $m$  grows, this subsequent noise addition may override the advantage of individual ranking with respect to multivariate microaggregation; in practice, this only happens in pretty extreme cases (with many attributes, small  $\varepsilon$  values and large  $k$  values), because multivariate microaggregation also incurs substantial information loss for large  $m$ . In Figure 3 we can see some such extreme cases ( $m = 13$ ,  $\varepsilon = 10$  and  $k > 80$  values) in which the method in [17] is able to match or slightly outperform our method; this is more noticeable for JSD than for RE because, in the former case, the discretization of the continuous attribute values into bins smooths the effects of the noise. It is however interesting to observe that, for  $\varepsilon = 10$ , our method is even able to outperform the multivariate microaggregation for  $k > 20$  (with 8 attributes) and for  $k > 30$  (with 13 attributes). This shows the greater room for improvement that the individual ranking offers over the multivariate microaggregation for  $m > 1$ .

In fact, by equating the noise scale parameter of [17] and our method, we have

$$n/k \times (\Delta(A_i)/(k \times \varepsilon)) = m \times (\Delta(A_i)/(k \times \varepsilon)).$$

Hence, both scale parameters coincide for  $n/k = m$ . Thus, for  $k > n/m$ , the scale parameter of [17] and therefore the noise added to reach  $\varepsilon$ -differential privacy are smaller. For the “Census” data set, this means  $k > (1,080/13) \approx$

83. When the number of attributes to be protected decreases to  $m = 8$ , we get  $k > (1,080/8) = 135$  (which is beyond the  $k$  values represented in the X-axes of the above figures); further, for  $m = 1$ , we get  $k > (1,080/1) = n$ , which is outright impossible. Moreover, the practical reduction of information loss is only noticeable for small  $\varepsilon$  values (0.1 and 1.0). For  $\varepsilon = 10.0$ , the high information loss produced by the insensitive MDAV algorithm alone severely limits the practical noise reduction of the previous work. For such  $\varepsilon$  value, our method significantly improves on the one in [17] for all  $k$ .

Finally, to evaluate the influence of the prior microaggregation algorithm, we measured the preservation of attribute variances as described in Section 4.2. This was done for “Census”, which is the only data set with continuous numeric attributes. The results with 8 attributes are reported in Table 1.

We can see that the variations of the *attribute variances* are always greater (for all attributes and all  $k$  and  $\varepsilon$  values) for the method in [17] than for the one proposed in this paper. Differences between the two methods are greater for larger  $\varepsilon$  values (*i.e.*, 1 and 10) and larger  $k$  values (*i.e.* 25 and above); in these cases, the amount of noise that needs to be added is smaller and, thus, the influence of the prior microaggregation is more noticeable. The differences between both methods illustrate how individual-ranking microaggregation does a better job at preserving the internal structure (and, thus the statistical properties) of the attributes, since these are aggregated independently. In contrast, multivariate microaggregation is more constrained because all the attributes of each record are considered at once; this suppresses more variance and hence incurs higher information loss. In any case, the variations of the attribute variances tend to decrease as  $k$  grows, which suggests that the prior microaggregation helps decrease the large variance introduced by the noise added to satisfy differential privacy. However, there are some cases (*i.e.* attributes 2, 4, 5 and 8) in which, for large  $\varepsilon$  and  $k$  values (*i.e.* 10 and 100, respectively), variations increase; this shows how, for relaxed values of  $\varepsilon$ , the distortion introduced by the coarser microaggregation dominates the reduction of noise.

## 5. Conclusions

In [17] a method was presented that combines  $k$ -anonymity and  $\varepsilon$ -differential privacy to reap the best of both models: namely, on the one side the reasonably low information loss incurred by  $k$ -anonymity and its lack of assumptions on data uses (which do not limit the kind of analyses that can be performed), and on the other side the robust privacy guarantees offered by

Table 1: “Census”: variations of attribute variances between the attributes in the original and masked data sets with the proposed method (IR, individual ranking) and the one in [17] (MV, MDAV multivariate microaggregation) using different values of  $k$  and  $\varepsilon$ .

	$k$	$\varepsilon = 0.1$		$\varepsilon = 1$		$\varepsilon = 10$	
		IR	MV	IR	MV	IR	MV
$\Delta(\sigma^2(A_1))$	2	25.89	25.06	22.84	25.01	9.42	24.77
	25	21.74	24.57	8.08	20.19	0.16	3.06
	50	20.75	23.05	2.33	9.80	0.02	0.45
	100	15.27	17.38	0.38	0.95	0.02	0.89
$\Delta(\sigma^2(A_2))$	2	9.14	8.21	7.40	8.20	3.45	8.12
	25	8.33	8.05	2.30	6.62	0.12	1.23
	50	7.41	7.52	1.13	3.63	0.04	0.12
	100	5.69	5.84	0.11	0.50	0.05	0.18
$\Delta(\sigma^2(A_3))$	2	24.97	24.90	11.79	22.89	0.47	10.36
	25	24.83	24.80	12.85	22.72	0.44	8.63
	50	24.17	24.83	9.64	22.56	0.37	8.10
	100	23.63	24.78	9.60	22.27	0.1	6.84
$\Delta(\sigma^2(A_4))$	2	10.46	9.58	8.69	9.55	3.72	9.46
	25	9.43	9.35	2.39	7.64	0.08	1.13
	50	8.29	8.78	1.16	3.81	0.00	0.06
	100	5.78	6.65	0.48	0.42	0.03	0.23
$\Delta(\sigma^2(A_5))$	2	16.71	15.87	14.50	15.84	6.59	15.62
	25	14.35	15.54	5.33	12.87	0.16	2.22
	50	13.13	14.60	2.41	6.63	0.01	0.06
	100	9.86	11.48	0.18	0.94	0.05	0.27
$\Delta(\sigma^2(A_6))$	2	22.07	21.22	19.30	21.21	8.20	21.04
	25	19.50	20.73	6.36	17.12	0.19	2.37
	50	15.61	19.44	3.63	8.05	0.03	0.15
	100	10.20	14.81	0.45	0.85	0.04	0.44
$\Delta(\sigma^2(A_7))$	2	8.61	7.69	6.91	7.69	3.12	7.57
	25	7.63	7.54	2.97	6.14	0.03	1.12
	50	6.57	7.05	0.95	3.25	0.02	0.06
	100	4.95	5.44	0.30	0.41	0.01	0.20
$\Delta(\sigma^2(A_8))$	2	70.22	69.71	64.01	69.66	24.06	68.89
	25	63.42	68.21	14.90	55.85	0.05	5.93
	50	49.28	64.05	3.66	25.56	0.12	0.25
	100	41.36	47.38	0.78	2.05	0.21	0.81

$\varepsilon$ -differential privacy. In this paper, we have offered an alternative method that, by relying individual ranking microaggregation, is able to effectively reduce even more the scale parameter of noise in most scenarios, which is of utmost importance for data analysis. Such noise reduction has been discussed theoretically and it has been illustrated empirically for two reference data sets, by focusing on the error introduced in the attribute values and the preservation of attribute distributions.

The method proposed here is also easier to implement than the one in [17], because the individual ranking algorithm only relies on the natural order of individual attributes. Moreover, its computational cost is  $O(n \times m)$ , whereas insensitive multivariate microaggregation takes  $O(n \times n)$ . Since usually  $n \gg m$ , the current method is more scalable as the number of records in a data set grows. Moreover, prior individual-ranking microaggregation incurs less information loss than the prior multivariate microaggregation used in [17]. Finally, the proposed method is especially indicated when only a subset of attributes needs to be protected (*e.g.* the confidential attributes).

We leave as future work the exploration of other types of noise. For instance, using a noise calibrated to the smooth sensitivity of the data would seem an interesting improvement. The main reason is that it would reduce the dependency of the amount of required noise on the size of the attribute domain.

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