On infinite chains of numerical semigroups

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Numerical semigroups are mathematical objects that appear in many different scenarios. In particular, they appear in algebraic geometry being the natural structure of Weierstrass semigroups and they appear in the theory of algebraic geometry codes, characterizing good families of codes, duality properties, and they are a useful tool to bound parameters such as the length, the correction capability or the dimension of codes.

A *numerical semigroup* is a submonoid of the additive monoid $\mathbf{N}$ with finite complement in $\mathbf{N}$. The cardinality and the greatest element of this complement are the *genus* and the *Frobenius number* of the numerical semigroup, respectively; the smallest non-zero element of the numerical semigroup is its *multiplicity*. We can recurrently construct a tree rooted in $\mathbf{N}$ containing all numerical semigroups so that the nodes at each level g of this tree are the numerical semigroups of genus g. In [3] Bras-Amorós and Bulygin characterized the numerical semigroups that have infinitely many descendants in this tree. Their criterion depends only on the elements of the numerical semigroup that are less than its Frobenius number.

In this talk we show that at each level of the tree of numerical semigroups there are more nodes with a finite number of descendants than the other way around. Furthermore, we find a self-replicating behavior in the subtrees formed by numerical semigroups with fixed multiplicity m. With this we can determine the number of numerical semigroups that have infinitely many descendants, genus g and multiplicity $m \leq 7$.

References

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